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OPTIMIZATION OF STABILITY OF A FLEXIBLE MISSILE UNDER FOLLOWER THRUST

Oleg N. Kirillov* and Alexander P. Seyranian**
The Lomonosov Moscow State University, Moscow, Russia

ABSTRACT

This paper addresses two formulations of a dynamic problem of structural optimization. We consider a beam moving in space under a tangential end force as an idealization of a flexible missile. This nonconservative system can lose stability under a certain critical end force either by flutter or by divergence. That depends on the mass and/or stiffness distributions of the beam.

We first consider a non-uniform beam supposing its cross-sections are similar geometric figures. We are searching for an optimal mass distribution of the beam with the constant volume constraint in the sense of maximization of the critical end force. In the second formulation of the problem we study a uniform beam carrying a nonstructural mass. In this case our goal is to find an optimal distribution of the nonstructural mass with constant volume constraint.

For the first problem the mass distribution of the beam with the critical flutter load $p_* \approx 290$ is obtained. In the second case it is shown with the use of Pontryagin's maximum principle that optimal solutions belong to the «bang-bang» type. Optimal distributions of nonstructural mass with two and four switching points are presented.

I. INTRODUCTION

The stability of a uniform beam moving in space under a tangential end force, as an idealization of a flexible missile, has been investigated first by Gopak [1], Feodosiev [2], Beal [3], and Goroshko [4]. The separated dimensionless differential equation of this problem describing transverse vibrations of the beam is

$$u^{IV} + p\left(u'(1-x)\right)' - \omega^2 u = 0 \qquad (1.1)$$

with the boundary conditions

$$u''\big|_{x=0} = 0\,,\; u'''\big|_{x=0} = 0\,,$$

$$u''\big|_{x=1} = 0\,,\; u'''\big|_{x=1} = 0\,, \qquad (1.2)$$

as shown in detail in [1-4]. Primes denote differentiation with respect to x. It has been found that this system loses stability by flutter under a critical end force $p_* = 109.69$ [2].

Sundararajan [5] considered the problem of optimal arrangement of nonstructural mass along such a beam. He plotted graphs showing dependence of the critical load on the displacement of the concentrated mass along the beam and found the optimal point. But this result seems to be not accurate since he used only two modes in Bubnov-Galerkin approximation. Sundararajan [5] also attempted to find an optimal continuous nonstructural mass distribution.

In this paper we use and develop further the ideas and methods presented in Seyranian and Sharanyuk [8], Pedersen and Seyranian [9], and Seyranian [10].

II. BASIC RELATIONS

Consider a flexible beam moving in space under a tangential end force with non-uniform cross-sections. It is assumed that the beam carries a nonstructural mass. This system is described by the following equations and boundary conditions [4]:

$$\begin{gathered} \tilde{m}\ddot{U} - \left(Q(s)U'\right)' + (EJU'')'' = 0 \\ Q(s) = -\frac{P}{M}\int_s^l \tilde{m}(\xi)d\xi, \\ EJU''\big|_{s=0} = 0, (EJU'')'\big|_{s=0} = 0, \\ EJU''\big|_{s=l} = 0, (EJU'')'\big|_{s=l} = 0, \end{gathered} \qquad (2.1)$$

where:

l is the length of the beam,

$M = \int_0^l \tilde{m}(\xi)d\xi$ - the total mass of the beam,

P - the follower force,

* Postgraduate-student, Department of Applied Mechanics, The Lomonosov Moscow State University.
** Professor, Institute of Mechanics, The Lomonosov Moscow State University.

$\tilde{m}(s) = \tilde{m}_1(s) + \tilde{m}_2(s)$,

$\tilde{m}_1(s)$ - the mass of the beam per unit length,

$\tilde{m}_2(s)$ is nonstructural mass per unit length,

and $U(s,t)$ is a deflection of the beam.

Suppose that cross-sections of the beam are similar geometric figures. Then we have:

$$J(s) = \frac{\tilde{m}_1^2(s) J_*}{\rho^2 S_*^2} = \gamma \tilde{m}_1^2(s), \qquad (2.2)$$

where ρ is the constant density of the beam, J_* is moment of inertia of a cross-section with the area S_* of the beam.

Let us take $U(s,t) = U(s)e^{i\Omega t}$ and introduce dimensionless variables:

$$x = \frac{s}{l},\ u = \frac{U}{l},\ m = \frac{\tilde{m}}{M} l,\ m_{1,2} = \frac{\tilde{m}_{1,2}}{M} l$$

$$p = \frac{Pl^4}{EM^2\gamma},\ \omega^2 = \frac{\Omega^2 l^5}{EM\gamma}. \qquad (2.3)$$

Now we get the separated dimensionless differential equation with the appropriate boundary conditions, which describe the eigenvalue problem for (2.1):

$$Lu = \left(m_1^2 u''\right)'' + p\left(u' \int_x^1 m(\xi) d\xi\right)' - m\omega^2 u = 0$$

$$\left(m_1^2 u''\right)\Big|_{x=0} = 0,\ \left(m_1^2 u''\right)'\Big|_{x=0} = 0, \qquad (2.4)$$

$$\left(m_1^2 u''\right)\Big|_{x=1} = 0,\ \left(m_1^2 u''\right)'\Big|_{x=1} = 0.$$

Further we shall consider two formulations of the problem (2.4), which differ by the function $m(x) = m_1(x) + m_2(x)$:

$$1)\ \ m = m_1(x),\ m_2 \equiv 0; \qquad (2.5)$$

$$2)\ \ m_1 = 1 - \kappa,\ m_2 = \kappa\mu(x), \qquad (2.6)$$

where $\int_0^1 \mu(\xi) d\xi = 1$. In particular, $\mu(x)$ can be a Dirac delta-function: $\mu(x) = \delta(x - a)$.

III. OPTIMIZATION PROBLEM FOR THE BEAM WITHOUT NONSTRUCTURAL MASS

Problem formulation

We first consider the beam without nonstructural mass. Combining (2.4) and (2.5), we obtain appropriate eigenvalue problem

$$Lu = \left(m^2 u''\right)'' + p\left(u' \int_x^1 m(\xi) d\xi\right)' - m\omega^2 u = 0,$$

$$\left(m^2 u''\right)\Big|_{x=0} = 0,\ \left(m^2 u''\right)'\Big|_{x=0} = 0,$$

$$\left(m^2 u''\right)\Big|_{x=1} = 0,\ \left(m^2 u''\right)'\Big|_{x=1} = 0. \qquad (3.1)$$

This nonconservative system can lose stability by flutter or by divergence depending on mass distribution $m(x)$ and load parameter p.

Now we formulate an optimization problem. For the system described by equations (3.1) we must find a mass distribution, satisfying the constant volume constraint, so as to obtain a maximum critical load p_*

$$\max_{m \in \Omega} p_*(m), \qquad (3.2)$$

$$\Omega = \left\{ m(x) : \int_0^1 m dx = 1; m(x) > 0, x \in [0,1] \right\}$$

For this purpose we use sensitivity analysis by introduction of the problem, adjoint to (3.1), and find a gradient function of the critical load parameter p_* with respect to the mass distribution $m(x)$. Then we formulate a variational principle which will be used for obtaining discretized problem for numerical solution.

Adjoint problem

If we multiply the first equation of (3.1) by the function $v(x)$, then integrate by parts the equality $\int_0^1 v(x) Lu(x) dx = 0$, and take into account the boundary conditions (3.1), we get the adjoint eigenvalue problem

$$L^* v = (m^2 v'')'' + p(v' \int_x^1 m(\xi) d\xi)' - m\omega^2 v = 0$$

$$(m^2 v'' + pv)\big|_{x=0} = 0, (m^2 v'' + pv)'\big|_{x=0} = 0,$$

$$(m^2 v'')\big|_{x=1} = 0, \qquad (m^2 v'')'\big|_{x=1} = 0. \quad (3.3)$$

With the use of the main (3.1) and adjoint (3.3) problems a «flutter condition» can be derived. Let ω_i, u_i be an eigenvalue and an eigenvector of the problem (3.1), and ω_j, v_j be an eigenvalue and an eigenvector of the problem (3.3). First we integrate twice by part the equalities

$$\int_0^1 v_j(x) Lu_i(x) dx = 0,\ \int_0^1 u_i(x) L^* v_j(x) dx = 0,$$

and subtract the second equality from the first.

Using the boundary conditions (3.1) and (3.3) we get the relation

$$\left(\omega_i^2-\omega_j^2\right)\int_0^1 mu_i v_j\,dx=0\,,$$

which for $\omega_i \neq \omega_j$ expresses the biorthogonality condition $\int_0^1 mu_i v_j\,dx=0$. At a flutter point the condition of a double eigenvalue, by arguments of continuity, gives the flutter condition

$$\int_0^1 mu_* v_*\,dx=0\,. \tag{3.4}$$

Gradient function

Now we can find the gradient function and study the sensitivity of the critical load parameter with respect to the mass distribution.

Let us take a variation $\delta m(x)$ of the mass distribution $m(x)$. Then the critical load parameter p_*, the critical frequency ω_*, and the eigenfunction u_* take increments $\delta p_*, \delta\omega_*, \delta u_*$. From equations (3.1) we get the equation and the boundary conditions in variations

$$\delta\left(Lu_*\right)=$$

$$=L\delta u_*-2\omega_*\delta\omega_* mu_*+\left(2m\delta mu_*''\right)''-\omega_*^2\delta mu_*+$$

$$+\delta p_*\left(u_*'\int_0^1 m(\xi)d\xi\right)'+p_*\left(u_*'\int_0^1 \delta m(\xi)d\xi\right)'=0\,, \tag{3.5}$$

$$\left.\left(2m\delta mu_*''+m^2\delta u_*''\right)\right|_{x=0}=0\,,$$

$$\left.\left(2m\delta mu_*''+m^2\delta u_*''\right)\right|_{x=1}=0\,,$$

$$\left.\left(2m\delta mu_*''+m^2\delta u_*''\right)'\right|_{x=0}=0\,,$$

$$\left.\left(2m\delta mu_*''+m^2\delta u_*''\right)'\right|_{x=1}=0\,. \tag{3.6}$$

Multiplying (3.5) from the left by v_*, then integrating with respect to x from 0 to 1, and using flutter condition (3.4), we obtain

$$\int_0^1 v_*\delta\left(Lu_*\right)dx=v_*\left(m^2\delta u_*''\right)'\Big|_0^1-v_*'\left(m^2\delta u_*''\right)\Big|_0^1+$$

$$+v_*\left(2m\delta mu_*''\right)'\Big|_0^1-v_*'2m\delta mu_*''\Big|_0^1+$$

$$+\int_0^1\left\{\delta m\left(2mu_*''v_*''-\omega_*^2u_*v_*-p_*v_*u_*'\right)+\right.$$

$$\left.+p_*v_*u_*''\int_x^1\delta m(\xi)d\xi+\delta p_* v_*\left(u_*'\int_x^1 m(\xi)d\xi\right)'\right\}dx=0,$$

where first four non-integral terms are zero by means of the boundary conditions in variations (3.6). Besides, the term

$$\int_0^1 p_*v_*u_*''\left(\int_x^1\delta m(\xi)d\xi\right)dx$$

can be transformed into

$$\int_0^1 p_*v_*u_*''\left(\int_0^1\delta m(\xi)d\xi\right)dx=\int_0^1\delta m(x)\left(p_*\int_0^x v_*u_*''d\xi\right)dx=$$

$$=\int_0^1\delta m(x)\left(p_*v_*(x)u_*'(x)-p_*v_*(0)u_*'(0)-p_*\int_0^x v_*'u_*'d\xi\right)dx.$$

Finally, we obtain the expression

$$\int_0^1 v_*\delta\left(Lu_*\right)dx=\int_0^1\left(2mu_*''v_*''-\omega_*^2u_*v_*-p_*v_*(0)u_*'(0)-\right.$$

$$\left.-p_*\int_0^x v_*'u_*'d\xi\right)\delta m dx+\delta p_*\int_0^1 v_*\left(u_*'\int_x^1 m(\xi)d\xi\right)'dx=0,$$

which can be rewritten in the following form

$$\delta p_*=\int_0^1 g_f\,\delta m dx\,, \tag{3.7}$$

$$g_f(x)=\frac{\omega_*^2u_*v_*-2mu_*''v_*''+p_*v_*(0)u_*'(0)+p_*\int_0^1 u_*'v_*'d\xi}{\int_0^1 v_*\left(u_*'\int_x^1 m(\xi)d\xi\right)'dx}.$$

The function $g_f(x)$ is the gradient function of the critical flutter load p_* with respect to the mass distribution $m(x)$. This function shows the sensitivity of flutter load p_* with respect to mass distribution. To compute $g_f(x)$ we have to know only solutions of the main (3.1) and adjoint (3.3) eigenvalue problems.

Under certain $m(x)$ and p_* system (3.1) can lose stability by divergence, i.e., when one of the eigenvalues ω becomes zero. The equations describing this situation can be derived from (3.1) when the equality $\omega=0$ is taken into consideration. Using the same technique, we obtain the gradient function $g_d(x)$ of the critical divergence load parameter p_* with respect to the mass distribution $m(x)$

$$g_d(x) = \frac{-2mu_*''v_*'' + p_*v_*(0)u_*'(0) + p_*\int_0^1 u_*'v_*'d\xi}{\int_0^1 v_*\left(u_*'\int_x^1 m(\xi)d\xi\right)' dx}. \tag{3.8}$$

Variational principle

Let us consider a functional $I(u,v)$, which is obtained from the scalar product (Lu,v) with the use of integration by parts

$$I(u,v) = \int_0^1 \Big\{m^2u''v'' - pv(0)u'(0) - pu'v'\int_x^1 m(\xi)d\xi - m\omega^2 uv\Big\}dx. \tag{3.9}$$

Variation of this functional with respect to u and v is

$$\delta I(u,v) = $$

$$= \int_0^1 \left\{(m^2u'')'' + p\left(u'\int_x^1 m(\xi)d\xi\right)' - m\omega^2 u\right\}\delta v dx +$$

$$+(m^2u'')\delta v'\Big|_{x=1} - (m^2v'')'\delta v\Big|_{x=1} -$$

$$-(m^2u'')\delta v'\Big|_{x=0} + (m^2u'')'\delta v\Big|_{x=0} +$$

$$+\int_0^1 \left\{(m^2v'')'' + p\left(v'\int_x^1 m(\xi)d\xi\right)' - m\omega^2 v\right\}\delta u dx +$$

$$+(m^2v'')\delta u'\Big|_{x=1} - (m^2v'')'\delta u\Big|_{x=1} -$$

$$-(m^2v'' + pv)\delta u'\Big|_{x=0} + (m^2v'' + pv)'\delta u\Big|_{x=0} = 0. \tag{3.10}$$

It is easy to see from (3.10) that stationarity of the functional $I(u,v)$ with respect to arbitrary smooth variations δu, δv is equivalent to the boundary value problems (3.1) and (3.3). The property of stationarity of the functional $I(u,v)$ in combination with Bubnov-Galerkin procedure or with a finite element method gives a simple way for discretization of our eigenvalue problems.

Discretization and method of solution

To obtain discrete approximation of the eigenvalue problems we use the property of stationarity of the functional $I(u,v)$. Let us take approximations

$$u(x) = \sum_{i=1}^N \alpha_i u_i(x),\ v(x) = \sum_{j=1}^N \beta_j v_j(x). \tag{3.11}$$

Substituting expansions (3.11) in (3.9) we get

$$I_N(\alpha,\beta) = \sum_{i=1}^N\sum_{j=1}^N \alpha_i\beta_j\Phi_{ji}, \tag{3.12}$$

$$\Phi_{ji} = \int_0^1 \Big\{m^2u_i''v_j'' - pv_j(0)u_i'(0) - pu_i'v_j'\int_x^1 m(\xi)d\xi - m\omega^2 u_iv_j\Big\}dx.$$

Stationarity conditions $\frac{\partial I_N}{\partial \alpha_i} = 0$, $\frac{\partial I_N}{\partial \beta_j} = 0$ written for the functional I_N lead to two systems of linear equations for determination of the coefficients α_i, β_j

$$\sum_{i=1}^N \Phi_{ji}\alpha_i = 0,\ j = 1..N, \tag{3.13}$$

$$\sum_{j=1}^N \Phi_{ji}\beta_j = 0,\ i = 1..N. \tag{3.14}$$

Note that the vectors $\alpha = (\alpha_1,..,\alpha_N)$ and $\beta = (\beta_1,..,\beta_N)$ are right and left eigenvectors of the matrix $\|\Phi_{ji}\|$, respectively. Equality $\mathbf{det}\|\Phi_{ij}\| = 0$ serves for finding the eigenvalues ω^2.

We use the following basis functions

$$u_1(x) = v_1(x) = 1,$$

$$u_2(x) = v_2(x) = x,$$

$$u_i(x) = v_i(x) = 1 + c_{1i}x + \sum_{k=2}^{i+1} c_{ki}x^{k+2},\ i = 3..N. \tag{3.15}$$

We choose basis functions $u_i(x)$, $v_i(x)$ satisfying the boundary conditions of the selfadjoint problem, which can be obtained from (3.1) or (3.3) when the condition $p = 0$ is taken into account. In addition, we demand the orthogonality of the basis functions with the different indexes

$$\int_0^1 mu_iu_jdx = 0,\ i \neq j.$$

With these two demands we can find coefficients c_{ki} in (3.15). Note that basis functions differ for different mass distributions. Thus we have to calculate u_i, v_i on each step of the optimization procedure.

Optimization procedure

To construct approximations to optimal mass distribution we use a gradient method.

Let us consider a variation of the critical load parameter p_* taking into account constant volume constraint $\int_0^1 \delta m(x)dx = 0$:

$$\delta p_* = \int_0^1 (g(x) - \varepsilon)\delta m dx, \varepsilon = const, \quad (3.16)$$

$g(x)$ can be either $g_f(x)$ or $g_d(x)$. If we choose variation of mass distribution as

$$\delta m = \alpha(x)(g(x) - \varepsilon), \quad (3.17)$$

where the gradient step $\alpha(x)$ is arbitrary nonnegative function, then in the first approximation we obtain

$$\delta p_* = \int_0^1 \alpha(x)(g(x) - \varepsilon)^2 dx \geq 0, \quad (3.18)$$

$$\varepsilon = \frac{\int_0^1 \alpha(x)g(x)dx}{\int_0^1 \alpha(x)dx}. \quad (3.19)$$

The inequality (3.18) shows that variation (3.17) causes monotonous growth of the functional of critical load for rather small step $|\alpha| << 1$, $x \in [0,1]$.

Each iteration of the optimization procedure consists of three steps.

First for current mass distribution we find critical load p_* and establish the mechanism of instability: flutter or divergence. For that we solve the main and the adjoint eigenvalue problems for different values of the load parameter p.

Then we calculate the appropriate gradient function $g_f(x)$ or $g_d(x)$ at the critical load p_* according to the expressions (3.7) or (3.8), respectively.

On the last step we obtain the variation of the mass distribution by (3.16) and calculate new mass distribution

$$m_{k+1} = m_k + \delta m, k = 0,1,2,.. \quad (3.20)$$

In our calculations we choose $m_0(x) \equiv 1$ as an initial approximation to the optimal mass distribution.

Numerical results

We start with the initial mass distribution $m(x) \equiv 1$. In this case the system loses stability by flutter under a critical load parameter $p_* = 109.69$ (Fig 1).

After several first iterations accompanied by the growth of p_* we get the mass distribution (Fig 2a) when the system loses stability by divergence (Fig 3a).

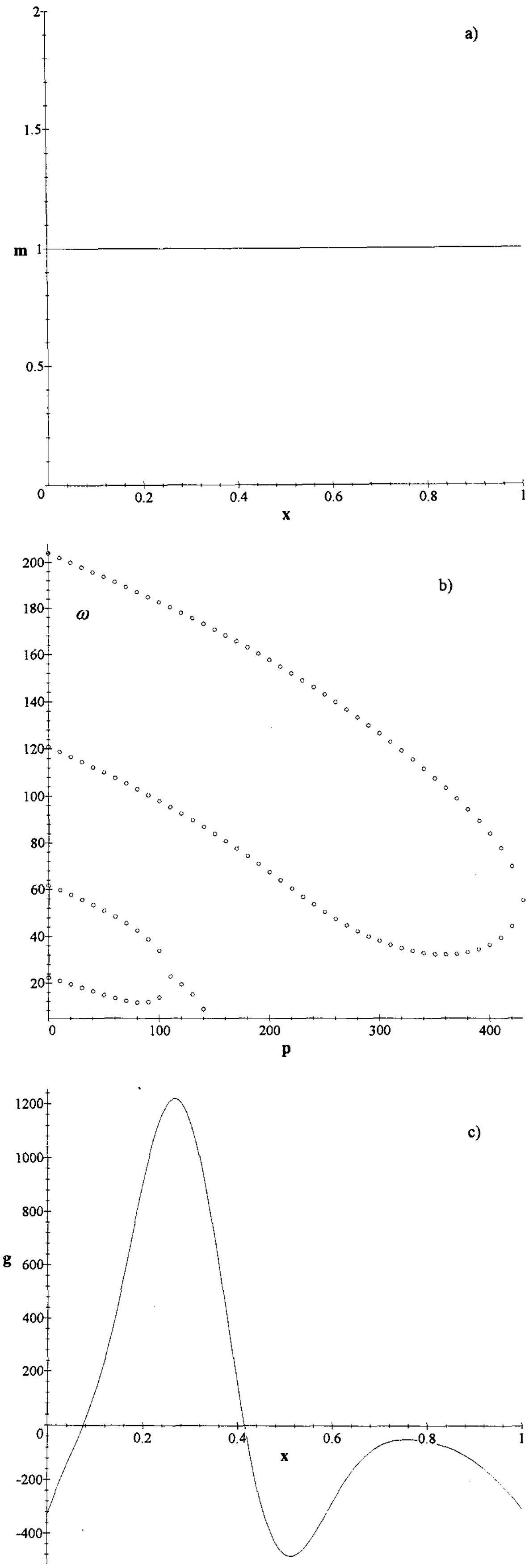

Figure 1. Initial mass distribution, characteristic curves, and gradient function

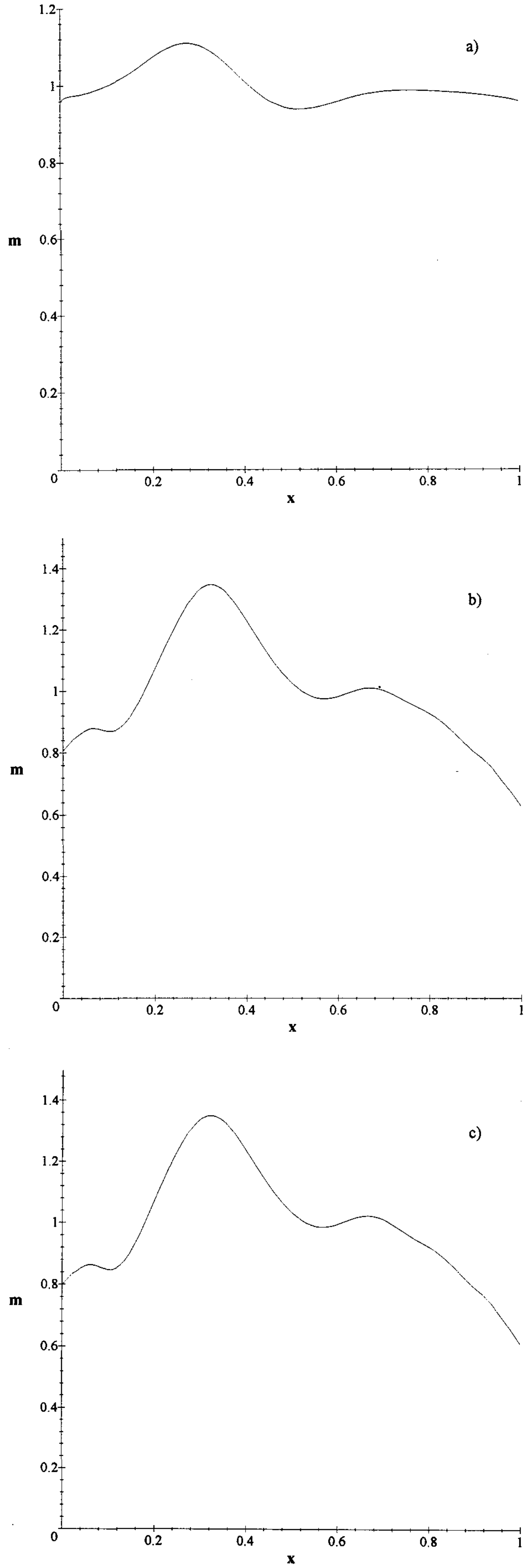

Figure 2. Evolution of mass distributions

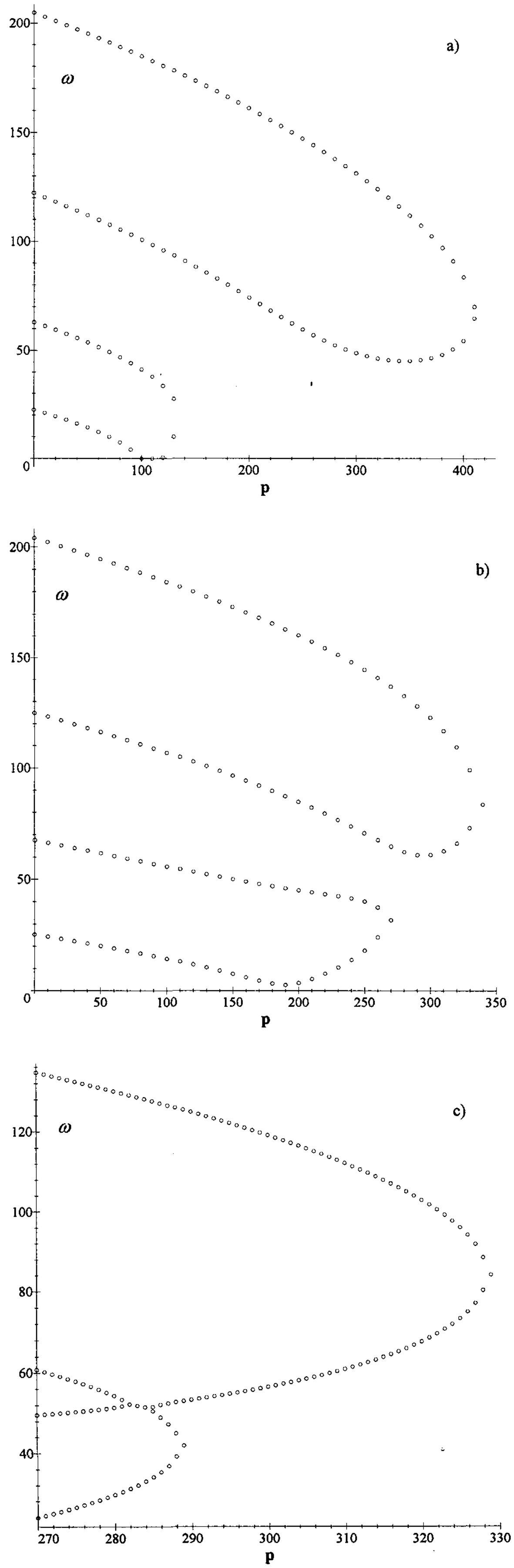

Figure 3 . Evolution of characteristic curves

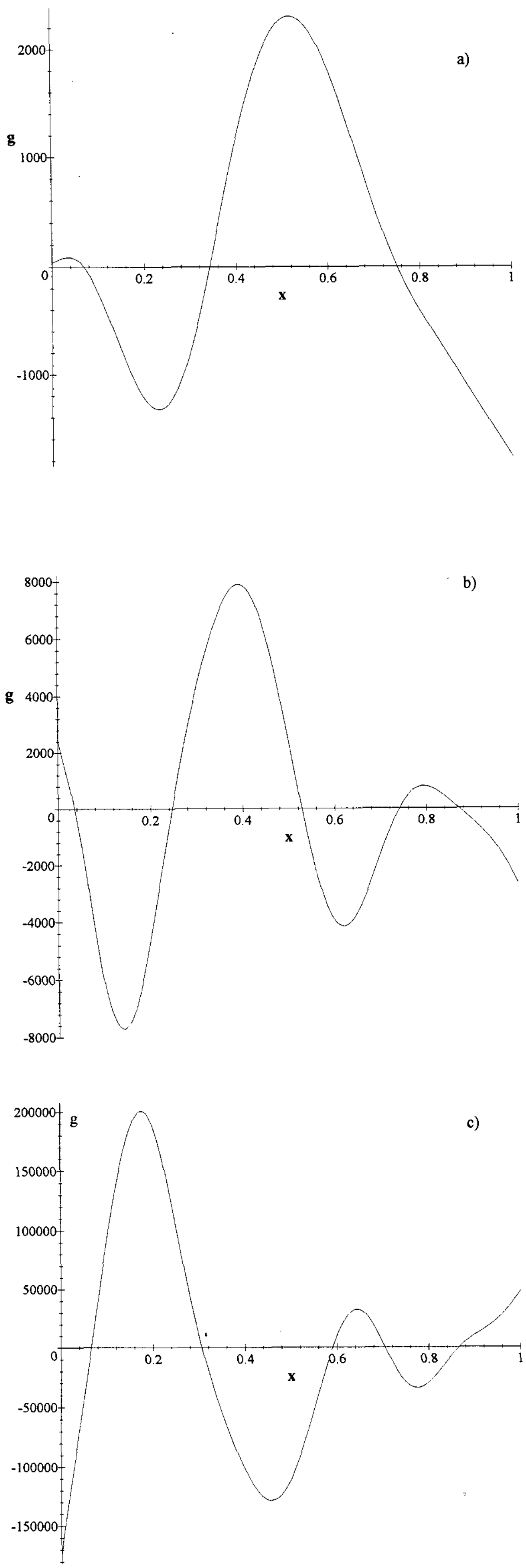

Figure 4. Evolution of gradient functions

In such situation we calculate gradient function of divergence critical load (Fig 4a) and use it for the mass distribution computation according to (3.17), (3.20).

So we are searching for the mass distribution which maximizes the minimal of the critical flutter or divergence loads. Some further iterations are shown in Figures 2 - 4. Finally we obtain the mass distribution with the critical flutter load $p_* \approx 290$ as shown in Figures 2c-3c.

In Fig 3c we also can see intersection of two curves describing dependence of frequencies of the system on the load parameter p. The appropriate mass distribution (Fig 2c) seems to be close to the optimal.

IV. OPTIMIZATION PROBLEM FOR THE BEAM CARRYING NONSTRUCTURAL MASS

Problem formulation

Now we consider the beam carrying nonstructural mass. According to (2.4) and (2.6) we have

$$(1-\kappa)^2 u^{IV} + p\left(u'\int_x^1 m(\xi)d\xi\right)' - m\omega^2 u = 0$$
$$u''\big|_{x=0} = 0, u'''\big|_{x=0} = 0,$$
$$u''\big|_{x=1} = 0, u'''\big|_{x=1} = 0, \tag{4.1}$$

where $m(x) = 1-\kappa+\kappa\mu(x)$, $\int_0^1 \mu(x)dx = 1$.

We denote κ, $0 \le \kappa < 1$, the part of nonstructural mass in the total mass of the system. The total mass of the system is equal to 1.

Let us denote $u_1 = u$, $u_5 = \int_x^1 m(\xi)d\xi$ and rewrite the system (4.1) in the normal form of differential equations of first order with the boundary conditions

$$u_1' = u_2\,, u_2' = u_3\,, u_3' = u_4\,,$$
$$(1-\kappa)^2 u_4' = -pu_3u_5 + \left(pu_2 + \omega^2 u_1\right)m\,,$$
$$u_5' = -m\,,$$
$$u_2(0) = u_2(1) = 0\,,$$
$$u_3(0) = u_3(1) = 0\,,$$
$$u_5(1) = 0\,. \tag{4.2}$$

This form is convenient to use the Pontryagin maximum principle [6].

Note that «control function» m (and also μ) appears in the right hand side of new system linearly. If additionally we introduce upper and lower constraints on $\mu(x)$ then we get from Pontryagin's maximum principle [6] that the optimal nonstructural mass distribution $\kappa\mu(x)$ will be of a «bang-bang» type.

Let $\frac{L}{\kappa}$ be the upper bound for $\mu(x)$ and 0 be the lower bound, respectively. Then we can formulate optimization problem.

For the system described by equations (4.1) we are searching for a nonstructural mass distribution $\kappa\mu(x)$, satisfying the constant volume constraint so as to obtain a maximum critical load p_*

$$\max_{\mu\in\Omega} p_*(\mu), \tag{4.3}$$

$$\Omega = \left\{ \mu(x): \int_0^1 \mu(x)dx = 1; 0 \le \kappa\mu(x) \le L, x \in [0,1] \right\}.$$

Adjoint problem

With the use of the technique described in section III we obtain the problem adjoint to (4.1)

$$\begin{gathered}(1-\kappa)^2 v^{IV} + p\left(v' \int_x^1 m(\xi)d\xi\right)' - m\omega^2 v = 0 \\ \left((1-\kappa)^2 v'' + pv\right)\Big|_{x=0} = 0, v''\big|_{x=1} = 0, \\ \left((1-\kappa)^2 v''' + pv'\right)\Big|_{x=1} = 0, v'''\big|_{x=1} = 0.\end{gathered} \tag{4.4}$$

Gradient function

Gradient function of the critical flutter load is

$$g_f(x) = \frac{\omega_*^2 u_* v_* + p_* v_*(0) u_*'(0) + p_* \int_0^x u_*' v_*' d\xi}{\int_0^1 v_* \left(u_*' \int_x^1 m(\xi)d\xi \right)' dx}. \tag{4.5}$$

For the critical divergence load we have

$$g_d(x) = \frac{p_* v_*(0) u_*'(0) + p_* \int_0^x u_*' v_*' d\xi}{\int_0^1 v_* \left(u_*' \int_x^1 m(\xi)d\xi \right)' dx}. \tag{4.6}$$

Both formulae (4.4), (4.5) are obtained using the method described in section III.

Variational principle.
Discretization and method of solution

Discrete approximation of main and adjoint eigenvalue problems is obtained using the property of stationarity of some functional as described earlier. The appropriate functional is

$$\begin{aligned} I(u,v) = \int_0^1 \Big\{ &(1-\kappa)^2 u''v'' - pv(0)u'(0) - \\ &- pu'v' \int_x^1 m(\xi)d\xi - m\omega^2 uv \Big\} dx. \end{aligned} \tag{4.7}$$

We use the same basis functions in Bubnov-Galerkin expansions as in (3.14).

Optimization procedure

To construct variation of nonstructural mass distribution we use method, which is described by Fedorenko in [7]. This method is convenient when solving the problems of optimal control with such constraints on control function as we have.

Let $\mu = (\mu_1, .., \mu_N)$ be a discrete approximation of the $\mu(x)$, $s = (s_1, .., s_N)$ be discrete approximation of the $\mu(x)$ variation, $F = \sum_{k=0}^{N-1} \left(s_{k+1} \int_{kh}^{(k+1)h} g(x) \right)$, and $h = \frac{1}{N}$ be the step of the discrete net.

Then for obtaining variation of mass distribution we must solve the linear programming problem

$$\max_{\{s_n\}} F, \ -\mu_n \le s_n \le \frac{L}{\kappa} - \mu_n, \sum_{n=1}^{N} s_n = 0. \tag{4.8}$$

Therefore for new nonstructural mass distribution we have

$$\kappa\mu_i^{(k+1)} = \kappa\mu_i^{(k)} + \alpha\kappa s_i^{(k)}, \ i = 1,..N, \tag{4.9}$$

where $\alpha \in [0,1]$ is chosen by researcher. Recall that distribution of the total mass of the system is $m(x) = 1 - \kappa + \kappa\mu(x)$.

As in the previous section each iteration of the optimization procedure consists of three steps.

First for current nonstructural mass distribution we find critical load p_* and establish the mechanism of instability: flutter or divergence. For that we solve the main (4.1) and the adjoint (4.4) eigenvalue problems for different values of the load parameter p.

Then we calculate the appropriate gradient function $g_f(x)$ or $g_d(x)$ at the critical load p_* according to the expressions (4.5) or (4.6), respectively.

On the last step we obtain the variation of the mass distribution solving the linear programming problem (4.8) and calculate new nonstructural mass distribution according to (4.9). As earlier we choose $\mu_0(x) \equiv 1$ as an initial approximation to the optimal nonstructural mass distribution.

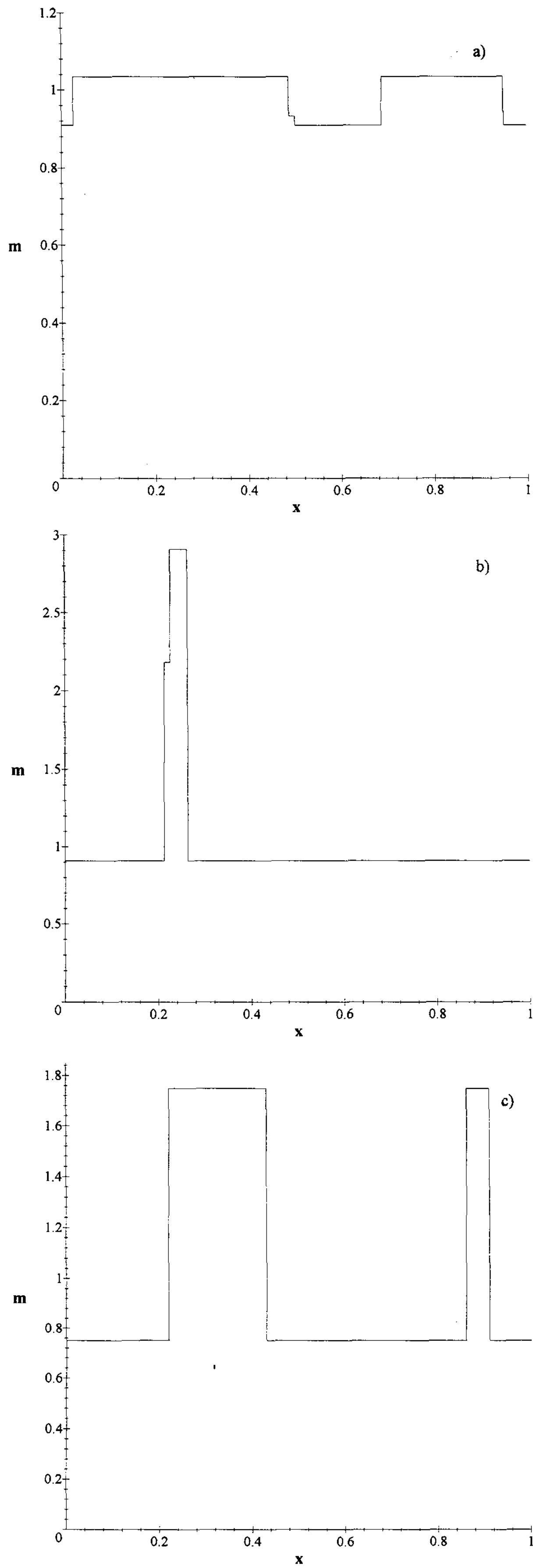

Figure 5. Mass distributions $m(x)$ for different values of parameters κ and L

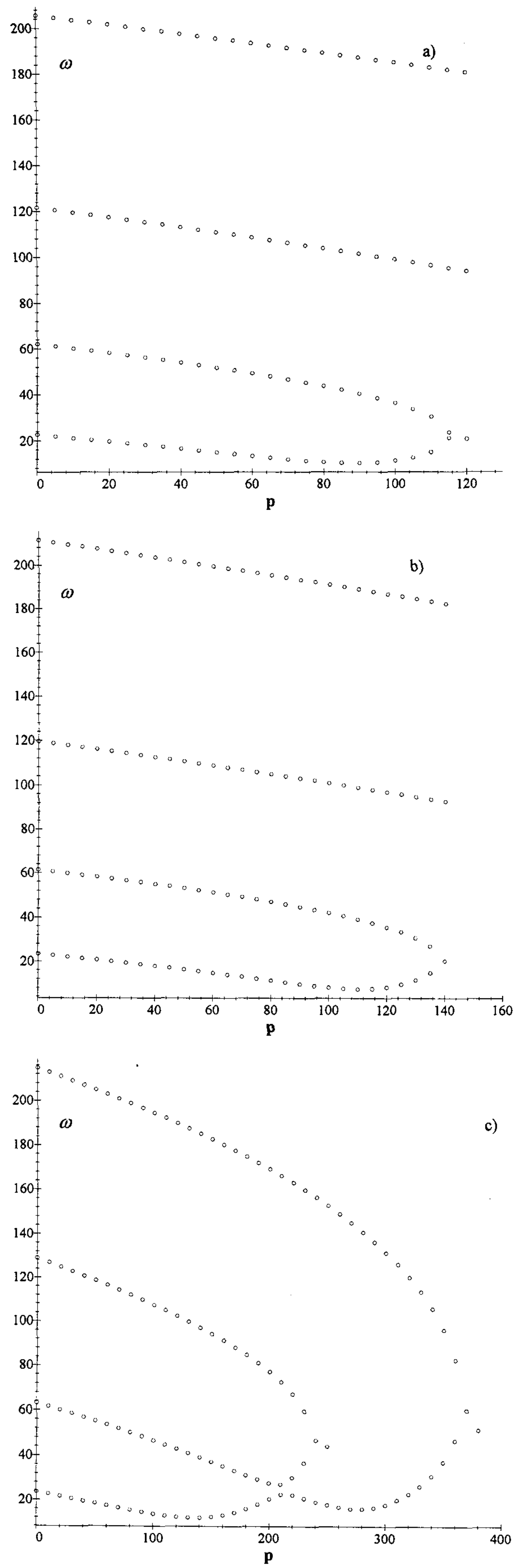

Figure 6. Characteristic curves for the mass distributions shown in Figure 5

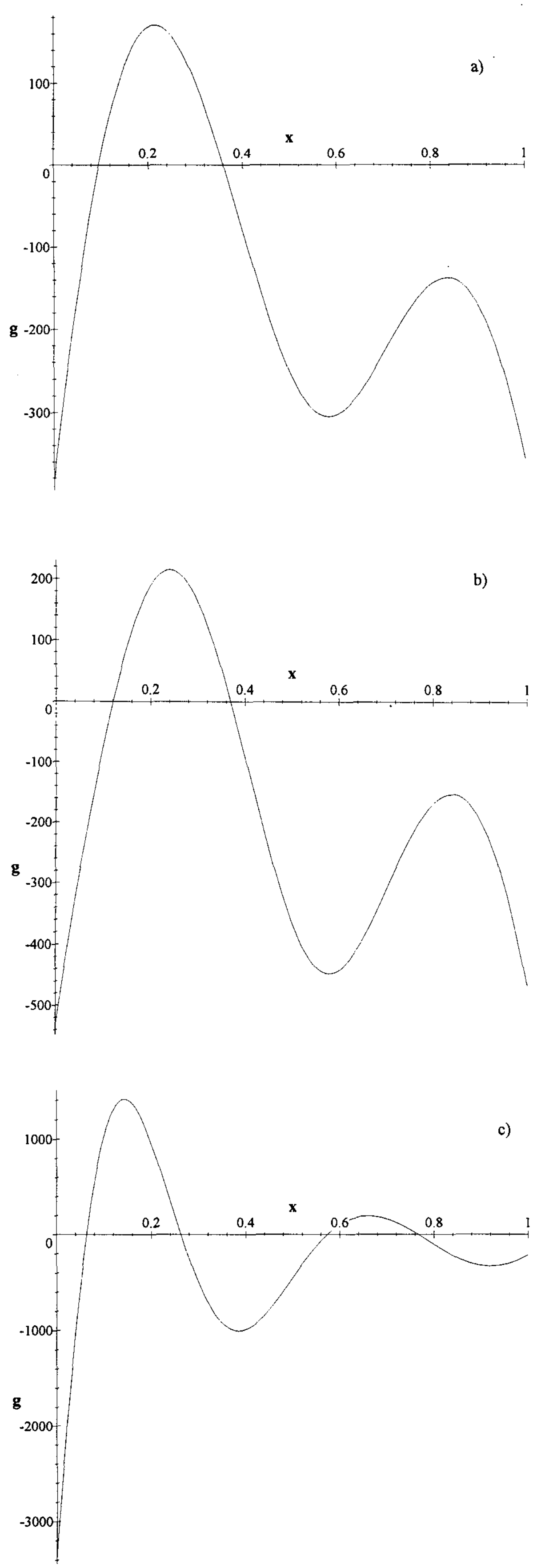

Figure 7. Gradient functions for the situations shown in Figure 6

Numerical results

In Figures 5-8 numerical results are presented. In Figures 5a-5b two optimal mass distributions are shown for $\kappa = \frac{1}{11}$ and different values of L. It can be seen that if the constraint L is not very large then optimal mass distribution has four switching points. Optimal mass distribution with two switching points appears when we take larger L. In Figure 5c the mass distribution with four switching points is shown for $\kappa = \frac{1}{4}$ and $L = 1$. In this case characteristic curves intersect similar to those shown in Figure 3c. Thus we can suppose that the mass distribution shown in figure 5c is close to optimal.

V. CONCLUSION

The paper is devoted to stability analysis and optimal distribution of stiffness and mass for a beam moving in space under a tangential end force. This problem is an idealization of dynamic stability of a flexible missile.

First we derived explicit expressions for gradient functions showing sensitivity of the critical flutter and divergence loads with respect to stiffness and mass rearrangements. For this purpose we introduced the adjoint eigenvalue problem. Then for stability analysis of the nonconservative problem we presented a variational principle which is very convenient for discretization of the continuous eigenvalue problem using Bubnov-Galerkin or other approximation techniques.

The numerical method based on the variational principle seems to be very effective for solving nonconservative stability problems many times which is necessary for finding optimal stiffness and mass distributions. Optimal solutions were obtained using gradient projection method and linear programming in the design space. The obtained results show that rational mass and stiffness distribution can radically improve stability characteristics of the moving beam.

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