Stabilization and destabilization in non-conservative gyroscopic systems

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Stability of a linear autonomous non-conservative system in presence of potential, gyroscopic, dissipative, and nonconservative positional forces is studied. The cases when the non-conservative system is close to a gyroscopic system or to a circulatory one, are examined. It is known that the marginal stability of gyroscopic and circulatory systems can be destroyed or improved up to asymptotic stability due to action of small non-conservative positional and velocity-dependent forces. The present contribution shows that in both cases the boundary of the asymptotic stability domain of the perturbed system possesses singularities such as "Dihedral angle" and "Whitney umbrella" that govern stabilization and destabilization. Approximations of the stability boundary near the singularities and estimates of the critical gyroscopic and circulatory parameters are found in an analytic form. In case of two degrees of freedom these estimates are obtained in terms of the invariants of matrices of the system. As an example, the asymptotic stability domain of the modified Maxwell-Bloch equations is investigated with an application to the stability problems of gyroscopic systems with stationary and rotating damping, such as the Cranndall gyroependulum, tippe top and Jellit’s egg. An instability mechanism in a system with two degrees of freedom, originating after discretization of models of a rotating disc in frictional contact and possessing the spectral mesh in the plane ‘frequency’ versus ‘angular velocity’, is described in detail and its role in the disc brake squeal problem is discussed.

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Asymptotic stability domain for the system (1) with 2 d.o.f. when \( \text{tr} \mathbf{K} > 0, \det \mathbf{K} > 0 \) and the sign of the left hand side in (7) is positive (a) or negative (b) and when \( \mathbf{K} < 0 \) (c); critical eigenvalue movement for the system (8) with 2 d.o.f. when \( \delta = \nu = 0 \) (d), and when \( \delta \neq 0 \) and \( \nu \neq 0 \) (e)-(g); bifurcation of the domain of asymptotic stability of the system (8) with 2 d.o.f. when \( \det \mathbf{D} \) is continuously changing from the positive values (h) to zero (i) and negative ones (j).

where \( \Omega_0 \) is the onset of the gyroscopic stabilization of system (3) and the real scalars \( d_1, d_2, n_1, \) and \( n_2 \) are calculated with the use of the generalized eigenvectors of the double eigenvalue \( \lambda = i\omega_0 \) of system (3) at \( \Omega = \Omega_0 \) [7]

\[
\begin{align*}
    d_1 &= \Re(\mathbf{u}_0^*\mathbf{D}\mathbf{u}_0), \quad d_2 = \Im(\mathbf{u}_0^*\mathbf{D}\mathbf{u}_1 - \mathbf{u}_1^*\mathbf{D}\mathbf{u}_0), \quad n_1 = \Im(\mathbf{u}_0^*\mathbf{N}\mathbf{u}_0), \quad n_2 = \Re(\mathbf{u}_0^*\mathbf{N}\mathbf{u}_1 - \mathbf{u}_1^*\mathbf{N}\mathbf{u}_0). 
\end{align*}
\]

In case of the two d.o.f. \( \mu = \Omega_0/2 \) and the formulas (4) are expressed directly by means of the matrices of the system (1) [3,4]

\[
\Omega_{cr}(\nu, \delta) = \Omega_0 + 2\Omega_0(\omega_0 \text{Tr} \mathbf{D})^{-2}(\nu/\delta - \beta_0)^2, \quad \beta_0 = (2\Omega_0)^{-1}\text{Tr} \left[ (\Omega_0^2 - \omega_0^2)! \mathbf{D} + \mathbf{K}\mathbf{D} \right].
\]

The equations (4) and (6) describe a surface with the singularity Whitney umbrella [1-7], which is seen in Fig. 1 (a)-(c), where the domain of asymptotic stability of the system (1) with 2 d.o.f. is shown. The domain bifurcates when \( \det \mathbf{D} \) passes through the threshold

\[
\det \mathbf{D} + (2\nu_f)^{-2}(k_{12}(d_{22} - d_{11}) - d_{12}(k_{22} - k_{11}))^2 = 0; \quad \nu_f = \sqrt{(\text{tr} \mathbf{K}/2)^2 - \det \mathbf{K}}.
\]

A special case of the system (1), occurring, for example, in the modeling of the squealing disk brake [6]

\[
\ddot{\mathbf{x}} + (2\Omega \mathbf{G} + \delta \mathbf{D})\mathbf{x} + ((\beta^2 - \Omega^2)I + \nu \mathbf{N})\mathbf{x} = 0,
\]

can be treated by the same method. Complicated singularities existing on the stability boundary of the system (8) (see Fig. 1 (h)-(j)) are responsible for the excursions of the eigenvalues to the right in the complex plane, causing the subcritical flutter instability, which is the reason for self-excited oscillations of a disc brake (squeal) [6].

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References