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Dissipation-induced subcritical flutter in the acoustics of friction

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We consider a gyroscopic system under the action of small dissipative and non-conservative positional forces, which has its origin in the models of rotating elastic bodies of revolution in frictional contact such as the singing wine glass or the squealing disc/drum brakes. The spectrum of the unperturbed gyroscopic system forms a spectral mesh in the plane 'frequency versus gyroscopic parameter' with double semi-simple purely imaginary eigenvalues at the nodes. In the subcritical range of the gyroscopic parameter the eigenvalues involved into the crossings have the same Krein signature and thus their splitting due to changes in the stiffness matrix, which break the rotational symmetry of the body, cannot produce complex eigenvalues and, therefore, flutter. We establish that perturbation of the gyroscopic system by the dissipative forces with the indefinite matrix can lead to the subcritical flutter instability even if the rotational symmetry is destroyed. With the use of the perturbation theory of multiple eigenvalues we explicitly find the linear approximation to the domain of the subcritical flutter, which turns out to have a conical shape. The orientation of the cone in the three dimensional space of the parameters, corresponding to gyroscopic, damping, and potential forces, is determined by the sign of an explicit expression involving the entries of both the damping and potential matrices. With the use of a time-dependent coordinate transformation we demonstrate that the conical zones of flutter for the original autonomous system coincide with the zones of the subcritical parametric resonance of the rotationally symmetric flexible body with the load moving in the circumferential direction.

Conical zones of the subcritical flutter instability induced by the indefinite damping

We consider a linear autonomous non-conservative gyroscopic system with dissipation

$$\ddot{\mathbf{x}} + (2\Omega\mathbf{G} + \delta\mathbf{D})\dot{\mathbf{x}} + (\mathbf{P} + \Omega^2\mathbf{G}^2 + \kappa\mathbf{K} + \nu\mathbf{N})\mathbf{x} = 0, \quad \mathbf{x} \in \mathbb{R}^{2n}, \tag{1}$$

where Ω is the gyroscopic parameter, $\mathbf{G} = \operatorname{diag}(\mathbf{J}, 2\mathbf{J}, \ldots, n\mathbf{J}) = -\mathbf{G}^T$ and $\mathbf{P} = \operatorname{diag}(\omega_1^2\mathbf{I}, \omega_2^2\mathbf{I}, \ldots, \omega_n^2\mathbf{I}) = \mathbf{P}^T$ are the matrices of gyroscopic and potential forces with

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{J} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \omega_1^2 < \omega_2^2 < \cdots < \omega_{n-1}^2 < \omega_n^2, \tag{2}$$

and the dissipative, conservative, and non-conservative perturbations with the real matrices $\mathbf{D} = \mathbf{D}^T$, $\mathbf{K} = \mathbf{K}^T$, and $\mathbf{N} = -\mathbf{N}^T$ are controlled by the parameters δ, κ, and ν, respectively. The transformation $\mathbf{x} = \mathbf{A}\mathbf{z} := \exp(-\Omega\mathbf{G}t)\mathbf{z}$ yields an equivalent to (1) potential system with the periodic perturbation, see [1]

$$\ddot{\mathbf{z}} + \delta\widetilde{\mathbf{D}}(t)\dot{\mathbf{z}} + (\mathbf{P} - \delta\Omega\widetilde{\mathbf{D}}(t)\mathbf{G} + \kappa\widetilde{\mathbf{K}}(t) + \nu\widetilde{\mathbf{N}}(t))\mathbf{z} = 0. \tag{3}$$

For $n = 2$ the matrix $\widetilde{\mathbf{N}}(t) := \mathbf{A}^{-1}\mathbf{N}\mathbf{A} = \mathbf{N}$ and the periodic stiffness and damping matrices are

$$\begin{aligned} 2\widetilde{\mathbf{K}}(t) := 2\mathbf{A}^{-1}\mathbf{K}\mathbf{A} &= \mathbf{I}\operatorname{tr}\mathbf{K} + (\mathbf{K} + \mathbf{J}\mathbf{K}\mathbf{J})\cos(2\Omega t) + (\mathbf{J}\mathbf{K} - \mathbf{K}\mathbf{J})\sin(2\Omega t), \\ 2\widetilde{\mathbf{D}}(t) := 2\mathbf{A}^{-1}\mathbf{D}\mathbf{A} &= \mathbf{I}\operatorname{tr}\mathbf{D} + (\mathbf{D} + \mathbf{J}\mathbf{D}\mathbf{J})\cos(2\Omega t) + (\mathbf{J}\mathbf{D} - \mathbf{D}\mathbf{J})\sin(2\Omega t). \end{aligned} \tag{4}$$

Equation (1) frequently originates after linearization and discretization of continuous models of rotating flexible bodies of revolution in frictional contact and describes their small oscillations in the stationary frame, whereas (3) describes them in the rotating frame [2–4]. Due to the rotational symmetry the eigenvalues ω_s^2, $s = 1, 2, \ldots, n$, of the matrix $\mathbf{P}$ are double semi-simple. The distribution of the doublets as a function of s is usually different for various bodies of revolution. For example, $\omega_s = s$ corresponds to the spectrum of free vibrations of a circular string [5].

Separating time by the substitution $\mathbf{x} = \mathbf{u}\exp(\lambda t)$ into (1), we arrive at the eigenvalue problem, whose eigenvalues for $\delta = \kappa = \nu = 0$ together with their complex conjugates $\lambda_s^{\pm} = i\omega_s \pm is\Omega$ and $\overline{\lambda_s^{\pm}} = -i\omega_s \mp is\Omega$ form a spectral mesh in the plane $(\Omega, \operatorname{Im}\lambda)$, see Fig. 1(a). The eigenvalues $\lambda_s^{\pm}$ correspond to the forward and backward traveling waves propagating in the circumferential direction of a rotating body of revolution, respectively, [3]. The sth backward traveling wave appears to be stationary when $\lambda_s^{\pm} = \overline{\lambda_s^{\pm}} = 0$ for the speeds of rotation $\pm\Omega_s^{cr} = \pm\omega_s/s$. We refer to the minimal of all Ω_s^{cr} as the critical speed and denote it Ω_{cr}. Then, the range $|\Omega| < \Omega_{cr}$ is called *subcritical* and the range $|\Omega| > \Omega_{cr}$ is called *supercritical* for the system (1). The eigenvalues $\overline{\lambda_s^{\pm}}$ in the supercritical range are associated with the reflected traveling waves [3].

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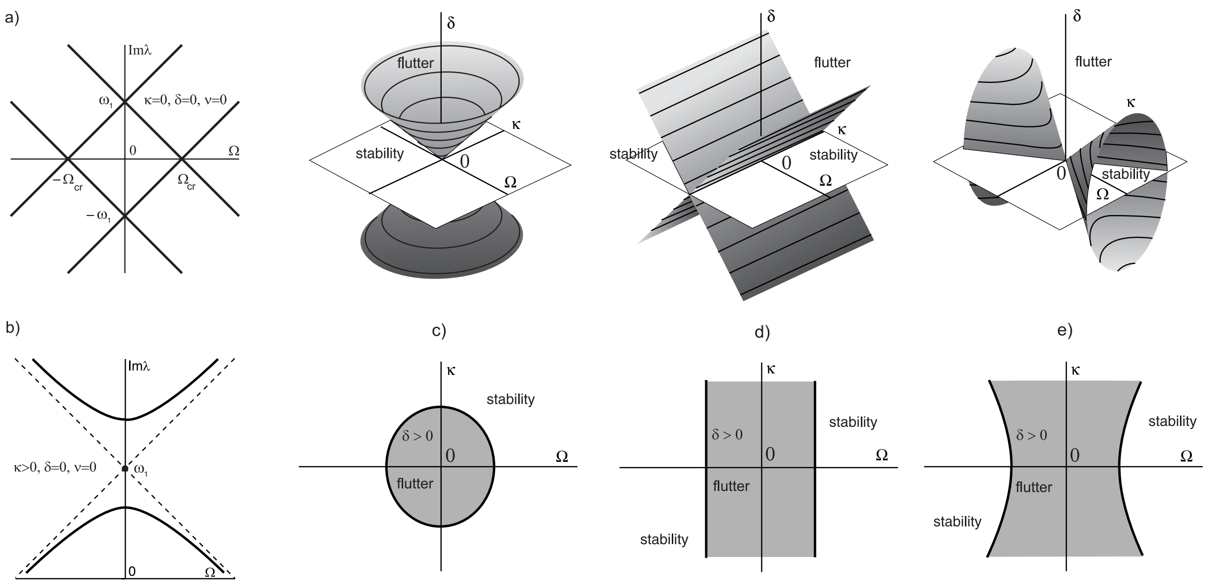

Fig. 1 n=2: (a) spectral mesh; (b) veering of the eigenvalue branches due to perturbation $\kappa\mathbf{K}$; domains of the subcritical flutter instability (parametric resonance) in the absence of the non-conservative positional forces ($\nu = 0$) for the idefinite matrix $\mathbf{D}$ with $\mathrm{tr}\mathbf{D} > 0$, $\det\mathbf{D} < 0$, and (c) $A > 0$, (d) $A = 0$, (e) $A < 0$.

Since the eigenvalues at the crossings in the subcritical range have the same Krein signature [6], they veer away under potential perturbation $\kappa\mathbf{K}$, destroying the rotational symmetry of the body, as shown in Fig. 1(b) for the case of $n = 2$ degrees of freedom. When, however, all types of forces are involved in the perturbation, then according to [5, 6] the real parts of the eigenvalues originated after the splitting of the double eigenvalue $i\omega_1$ for $n = 2$ are $\mathrm{Re}\lambda = (-\mathrm{tr}\mathbf{D}/4 \pm \sqrt{|c| + \mathrm{Re}c})\delta/4$ with

$$\mathrm{Re}c = \left(\frac{\mu_1 - \mu_2}{4}\right)^2 \delta^2 - \left(\frac{\rho_1 - \rho_2}{4\omega_1}\right)^2 \kappa^2 - \Omega^2 + \frac{\nu^2}{4\omega_1^2}, \quad \mathrm{Im}c = \frac{\Omega\nu}{\omega_1} - \delta\kappa\frac{2\mathrm{tr}\mathbf{KD} - \mathrm{tr}\mathbf{K}\mathrm{tr}\mathbf{D}}{8\omega_1}, \tag{5}$$

where $\mu_{1,2}$ and $\rho_{1,2}$ are the eigenvalues of the matrices $\mathbf{D}$ and $\mathbf{K}$, respectively. For $\nu = 0$ the condition $\mathrm{Re}\lambda < 0$ yields the linear approximation to the domain of asymptotic stability in the space of the parameters δ, Ω, and κ

$$\delta\mathrm{tr}\mathbf{D} > 0, \quad \kappa^2 A + \Omega^2(2\omega_1\mathrm{tr}\mathbf{D})^2 > -\det\mathbf{D}(\omega_1\mathrm{tr}\mathbf{D})^2\delta^2. \tag{6}$$

For the damping matrices $\mathbf{D} > 0$ the conditions (6) are always fulfilled, whereas for the indefinite damping matrices with $\det\mathbf{D} < 0$ from (6) follow the expressions for the flutter instability domain, which has a form of the half of a cone for $A := \det\mathbf{D}(\rho_1 - \rho_2)^2 + (k_{12}(d_{22} - d_{11}) - d_{12}(k_{22} - k_{11}))^2 > 0$, the dihedral angle for $A = 0$, and the domain adjacent to a half of a cone for $A < 0$, see Fig. 1(c-e). The orientation of the instability domain is determined also by the Krein signature of the eigenvalues involved into the corresponding crossing, which is substantially different in the subcritical and in the supercritical regions [6, 7]. In the plane (Ω, κ) for a fixed $\delta > 0$ the instability domain has, respectively, the form of an ellipse, a stripe, or a region contained between the branches of a hyperbola. The latter case shows that a widely known in the engineering practice approach to the squeal suppression by reducing the rotational symmetry of the rotor is not efficient in the presence of indefinite damping, which originates from the brake pads with the negative friction-velocity gradient [3, 4].

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References

[1] V. V. Kozlov, Gyroscopic stabilization and parametric resonance. J. Appl. Maths. Mechs. **65**(5), 715 (2001).
[2] R. Hryniv, P. Lancaster, Stabilization of gyroscopic systems, Z. Angew. Math. Mech. **81**(10), 675 (2001).
[3] J. E. Mottershead, Vibration- and friction-induced instabilities in discs. The Shock and Vibration Digest. **30**(1), 14 (1998).
[4] H. Ouyang, Parametric vibration induced by moving loads, in: Disc brake squeal: mechanism, analysis, evaluation, and reduction/prevention. [ed.] F. Chen, C.A. Tan, R.L. Quaglia, (SAE, Warrendale, 2006) pp. 49-78.
[5] O. N. Kirillov, How to Play a Disc Brake: A Dissipation-Induced Squeal. SAE Paper 2008-01-1160, In: Noise and Vibration, 2008, SP-2158, SAE World Congress and Exhibition, Detroit, MI, USA. (SAE, Warrendale, 2008) pp. 167–180.
[6] O.N. Kirillov, Subcritical flutter in the acoustics of friction. Proc. R. Soc. A **464**, (2008). doi: 10.1098/rspa.2008.0021
[7] O.N. Kirillov. Perturbation of multiparameter non-self-adjoint boundary eigenvalue problems for operator matrices. Preprint arXiv:0803.2248v2 [math-ph] 26 Mar 2008.

