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## Erratum

## Erratum to “Brouwer’s problem on a heavy particle in a rotating vessel: Wave propagation, ion traps, and rotor dynamics” [Phys. Lett. A 375 (2011) 1653]

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In this Letter a number of inaccuracies have occurred:

(1) Although it is true that the singularity of the domain of marginal stability at the origin of the  $(k_1, k_2, \Omega)$  space is related to zero eigenvalue of algebraic multiplicity 4, the Jordan structure of this eigenvalue reported in the Letter as  $0^4$ , which corresponds to a single quadruplet of geometric multiplicity 1, is not correct. Indeed, at  $k_1 = k_2 = 0$  the surface is flat and at  $\Omega = 0$  it is non-rotating. Hence, the coordinates of the perturbed particle will change linearly with time. Since the equations for  $x$  and  $y$  are decoupled at these values of parameters, the spectrum consists of two zero eigenvalues of algebraic multiplicity 2 and geometric multiplicity 1. Therefore, the type of the singularity is  $0^2 0^2$ . The shape of the singular surface shown in Fig. 3 only reminds that of the classical Swallowtail that in generic Hamiltonian systems has codimension 3 and corresponds to the quadruple pure imaginary eigenvalue of geometric multiplicity one,  $(i\omega)^4$ , see Galin [11]. The singularity  $0^2 0^2$  has codimension 3 in the Brouwer’s problem owing to an additional symmetry.

With this, the paragraph at the end of the Introduction should read as:

“At the origin, the conditions  $k_1 = k_2 = 0$  and  $\Omega = 0$  yield two double zero eigenvalues of geometric multiplicity 1. The shape of the stability boundary near the singularity  $0^2 0^2$  at the origin reminds that of the classical Swallowtail singularity,  $(i\omega)^4$ , that corresponds to a quadruple pure imaginary eigenvalue of geometric multiplicity one in generic Hamiltonian systems [11]. The Swallowtail-like singularity  $0^2 0^2$  has codimension 3 in the Brouwer’s problem that is described by the three parameter equivariant

Hamiltonian system (1). We note that in the works [9,10] the Swallowtail singularity  $(i\omega)^4$  had been found on the stability boundary of a two-link rotating shaft that has twice as many degrees of freedom. The Brouwer’s problem on a heavy particle in a rotating vessel appears to be the simplest physically meaningful system that possesses such a complicated shape of the stability boundary and as we will show further in the text, this fundamental paradigmatic model has deep connections to many important effects of classical and modern physics.”

The sentence at the end of page 1654 should read as: “Moreover, we discover that in the  $(k_1, k_2, \Omega)$ -space the full stability domain given by inequalities (3) is bounded by a surface that has a Swallowtail-like singularity at the origin, see Fig. 3.”

In Fig. 3 at the origin there should be “ $0^2 0^2$ ” instead of “ $0^4$ ”.

The caption to Fig. 3 should read as: “The Swallowtail-like singularity at the origin on the stability boundary of the Brouwer’s problem on a heavy particle in a rotating vessel illustrating peculiarities of gyroscopic stabilization.”

In Conclusion, the fourth sentence should read as: “We established new results on stability in the Brouwer’s problem that include discovery of the Swallowtail-like singularity on the stability boundary of the undamped Brouwer’s equations.”

In the Abstract, the fourth sentence should read as: “In particular, we find that the boundary of the stability domain of the undamped Brouwer’s problem possesses the Swallowtail-like singularity corresponding to a couple of double zero eigenvalues.”

(2) The sentence containing formula (15) should read as:

“Assuming  $A = u_1 + iu_2$ , linearizing the non-linear partial differential equation (14) about the basic traveling wave solution with the amplitude  $\mathbf{u}_0 = (u_1^0, u_2^0)$ , spacial wavenumber  $k$  and the frequency  $\omega$  and then expanding the periodic in  $x$  solution with the wavenumber  $\sigma$  of the linearized problem into Fourier series, we

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find that the  $\sigma$ -dependent modes decouple into four-dimensional subspaces for each harmonic with the number  $n$ , so that for  $n = 1$  we get

$$\begin{aligned} \mathbf{J}\dot{\mathbf{v}} + 2\alpha k\sigma\mathbf{J}\mathbf{w} - \alpha\sigma^2\mathbf{v} + 2\gamma\mathbf{u}_0\mathbf{u}_0^T\mathbf{v} &= 0, \\ \mathbf{J}\dot{\mathbf{w}} - 2\alpha k\sigma\mathbf{J}\mathbf{v} - \alpha\sigma^2\mathbf{w} + 2\gamma\mathbf{u}_0\mathbf{u}_0^T\mathbf{w} &= 0, \end{aligned} \quad (15)$$

where dot indicates time differentiation, the matrix  $\mathbf{J}$  is defined in (8), the dyad  $\mathbf{u}_0\mathbf{u}_0^T$  is a  $2 \times 2$  symmetric matrix and  $\omega = \alpha k^2 - \gamma\|\mathbf{u}_0\|^2$ ."

(3) The formula (17) should read as: " $\mathbf{J}\mathbf{B} = -q\mathbf{J} - \gamma\mathbf{D}$ ."

These corrections have no further impact on the content of the Letter.