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Unfolding of eigenvalue surfaces near a diabolic point due to a complex perturbation. (English)

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The energy surfaces of quantum physical systems can cross each other. For instance, they can form two sheets of a double cone known as 'diabolo', the apex of the cone being called a diabolic point. The energy surfaces are described by eigenvalues of of real symmetric or Hermitian operators depending on a set of parameters. The diabolic point is a point where the eigenvalues coalesce while corresponding eigenvectors remain linearly independent. Another kind of singularities, exceptional point, is a point where both eigenvalues and eigenvectors coalesce forming a Jordan block. Both types of points were observed experimentally. The diabolic point can bifurcate under arbitrary complex perturbations leading to topological singularities of eigenvalue surfaces such as a 'double coffee filter' with two exceptional points or a 'diabolic circle' of exceptional points.
In the present paper, the effects of complex perturbations in multiparameter families of real symmetric and Hermitian matrices are studied. The general theory of coupling of eigenvalues for complex matrices of arbitrary dimantion smoothly depending on multiple real parameters developed earlier in *A. P. Seyranian, O. N. Kirillov* and *A. A. Mailybaev* [J. Phys. A: Math. Gen. 38, 1723-1740 (2005; Zbl pre02145319)] is used. In the case of real symmetric matrices, the unfolding of eigenvalue surfaces near a diabolic p[oint under real and complex perturbations is studied. The origination of singularities such as a 'double coffee filter' and a 'diabolic circle' is analytically described. Unfolding of a diabolic point of a Hermitian matrices under an arbitrary complex perturbation is analytically treated as well. The theory presented requires only eigenvectors and derivatives of the matrix taken at the singular point while the size of the matrix and its dependence on the parameters can be arbitrary. Explicit asymptotic expressions for the surfaces of refractive indices in crystal optics are obtained to illustrate the theory.

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Keywords : coupling of eigenvalues; complex perturbation of matrices; unfolding of eigenvalue surfaces; diabolic point; exceptional point

Classification :

∗81Q15 Perturbation theories for operators and differential equations
47A75 Eigenvalue problems (linear operators)
15A18 Eigenvalues of matrices, etc.
78A10 Physical optics

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