MR2313764 (Review) 70J25 (70E18 70E50)
Kirillov, Oleg N. (D-DARM-MCE)
Destabilization paradox due to breaking the Hamiltonian and reversible symmetry. (English summary)
Internat. J. Non-Linear Mech. **42** (2007), *no. 1,* 71–87.

The second-order two-dimensional differential system

$$\frac{dx^2}{dt^2}+(\Omega G+\delta D)\frac{dx}{dt}+(K+\nu N)x=0,$$

where $K=K^\intercal$, $D=D^\intercal$, $G=-G^\intercal$ and $N=-N^\intercal$, is regarded here first as a perturbation of the imperfect circulatory (reversible) system

$$\frac{dx^2}{dt^2}+(K+\nu N)x=0$$

(that is, when $\delta,\Omega\ll\nu$) and second as a perturbation of the imperfect gyroscopic (Hamiltonian) system

$$\frac{dx^2}{dt^2}+\Omega G\frac{dx}{dt}+Kx=0.$$

The analysis reveals the surprising fact that the boundary (surface) of the asymptotic stability regions of the perturbation possesses singularity features of the types studied, e.g., by Arnold (dihedral and trihedral angles and the Whitney umbrella). The complex destabilization paradox due to Ziegler is also carefully disclosed here. In the first case, it is established that the magnitude of the non-conservative positional force at the onset of flutter with vanishingly small dissipative and gyroscopic forces does not exceed that of the circulatory system, thus undergoing a jump in the critical load which is characteristic of the destabilization paradox in Ziegler's form. Simple estimates of the jumps in the critical load are also given:

$$\nu_f\mp\nu_0^{\mp}(\beta)=\nu_{\mathrm{f}}\frac{2}{(\mathrm{tr}\,D)^2}(\beta\mp\beta_*)^2+o((\beta\mp\beta_*)^2),$$

where

$$\beta=\frac{\Omega}{\delta},\quad \beta_*=2\nu_{\mathrm{f}}^{-1}\mathrm{tr}(K-\omega_f^2I)D,\quad \omega_{\mathrm{f}}=\sqrt{\frac{1}{2}\mathrm{tr}\,K}$$

and

$$\nu_0^{\pm}(\beta)=\nu_f\frac{4\beta\beta_*}{[(\mathrm{tr}\,D)^2+4\beta^2]^{-1}}\pm\sqrt{(\mathrm{tr}\,D)^2+4(\beta^2-\beta_*^2)}.$$

A finer analysis is performed based on a perturbation technique for eigenvalues. It shows that a conservative system with $K>0$ can be made asymptotically stable by introducing small gyroscopic and damping forces with semi-definite or indefinite matrix D satisfying the condition

$$0>\det D\geq-(4\nu_f^2)^{-1}\left(k_{12}(d_{22}-d_{11})-d_{12}(k_{22}-k_{11})\right)^2.$$

If this condition does not hold, the asymptotic stability can be reached only in the presence of gyroscopic, damping and circulatory forces. In the second case, when $K > 0$ and D satisfies the previous condition, the domain of asymptotic stability in the (δ, ν, Ω)-space ($\Omega \neq 0$) has the form of a dihedral angle with the Ω-axis as its edge. By increasing $|\Omega|$, we run into a gyroscopic destabilization phenomenon. When $K < 0$, the gyroscopic stabilization of the statically unstable conservative system can be improved up to asymptotic stability by small damping and circulatory forces, if the magnitudes of these forces are in the narrow region with the boundaries depending on Ω. The critical values are also estimated:

$$\Omega_{\rm cr}^{\pm} = \pm\Omega_0^{+} \pm \Omega_0^{+} 2(\omega_0 \operatorname{tr} D)^{-2}(\gamma \mp \gamma_*)^2 + o((\gamma \mp \gamma_*)^2),$$

where

$$\nu = \gamma\delta, \quad \Omega_0^{+} = \sqrt{-\operatorname{tr} K + 2\sqrt{\det K}}, \quad \omega_0 = \sqrt[4]{\det K}$$

and

$$\gamma_* = (2\Omega_0^{+})^{-1} \operatorname{tr}(K + ((\Omega_0^{+})^2 - \omega_0^2)D).$$

Again, the critical value of the gyroscopic parameter Ω generally undergoes a jump ($\Omega_{\rm cr}^{+}(\pm\gamma_*) = \pm\Omega_0^{+}$) up for infinitely small δ, ν, which is characteristic of the destabilization paradox in Ziegler's form.

Therefore, in the presence of small damping and non-conservative positional forces, gyroscopic forces can both destabilize a statically stable conservative system and stabilize a statically unstable conservative system. The author validates his theoretical conclusions by a detailed analysis of the modified Maxwell-Bloch equations ($D = I$, $K = \kappa I$): the Whitney umbrella is present in the stability analysis outcome of the Crandall gyropendulum, the rising egg and the tippe top motion. The paper is clearly written and a lot of fundamental and novel references are given and commented upon, so people working in dynamical systems and mechanics will find it quite interesting.

Reviewed by *Octavian G. Mustafa*