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Collapse of the Keldysh chains and stability of continuous nonconservative systems. (English summary)
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The authors of this paper investigate the bifurcation of multiple eigenvalues of non-self-adjoint linear differential operators acting on the interval [0,1] with regular boundary conditions. They consider the general situation where both the coefficients of the operator and the boundary form depend smoothly on a vector of real parameters. Although the results presented hold for eigenvalues with the Keldysh chain of arbitrary length, most of the thorough analysis is made either for the semi-simple case or for the case where the length of this chain does not exceed 2. Recall that Keldysh chains are the analogues of Jordan chains of vectors for non-self-adjoint operators.

The main achievements of this well written paper may be summarized as follows. The authors compute, in quite an explicit manner, the expansion of the eigenvalues and eigenfunctions of the corresponding problem along smooth curves of the parameter space. This formula is given in terms of the derivatives of the differential expression and those of the boundary form with respect to the parameter, as well as the functions of the Keldysh chain evaluated at the point where the multiple eigenvalue occurs. They apply this result to describing properties of the stability boundaries of general circulatory systems. As a particular example, they test their result by analyzing the behaviour of eigenvalues in the vicinity of the stability boundary for the extended Beck column problem.

Reviewed by *Lyonell S. Boulton*

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