Summary: The spectrum of the spherically symmetric $\alpha^2$-dynamo is studied in the case of idealized boundary conditions. Starting from the exact analytical solutions of models with constant $\alpha$-profiles, a perturbation theory and a Galerkin technique are developed in a Krein space approach. With the help of these tools, a very pronounced $\alpha$-resonance pattern is found in the deformations of the spectral mesh as well as in the unfolding of the diabolical points located at the nodes of this mesh. Non-oscillatory as well as oscillatory dynamo regimes are obtained. An estimation technique is developed for obtaining the critical $\alpha$-profiles at which the eigenvalues enter the right spectral half-plane with non-vanishing imaginary components (at which overcritical oscillatory dynamo regimes form). Finally, Fréchet derivative (gradient) based methods are developed, suitable for further numerical investigations of Krein space related setups such as MHD $\alpha^2$-dynamos or models of $PT$-symmetric quantum mechanics.

Keywords: $\alpha^2$-dynamo; perturbation theory; Galerkin technique; diabolical points; Fréchet derivative methods

Classification:

* 47N50 Appl. of operator theory in quantum physics
  47B50 Operators on a space with an indefinite metric
  46C20 Spaces with indefinite inner product
  47A11 Local spectral properties
  32S05 Local singularities (analytic spaces)
  81Q15 Perturbation theories for operators and differential equations
  82D10 Plasmas