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Overlapping of Characteristic Curves and Optimization of Nonconservative Systems

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Summary An asymptotic theory of overlapping of characteristic curves in multi-parameter circulatory systems is presented. An analytical description of the disappearance effect in general multi-parameter non-conservative systems is done. The results obtained are useful in the problems of structural optimization with stability criteria, since they allow one to fin new necessary optimality conditions.

Statement of the problem and main results

We consider a nonconservative system governed by the equation $\ddot{\mathbf{y}} + \mathbf{A}\mathbf{y} = 0$, where $\mathbf{A}$ is a real nonsymmetric $m\times m$ matrix smoothly dependent on a vector of parameters $\mathbf{p}\in R^n$. At the fixed $\mathbf{p}$ this system is stable if and only if all the eigenvalues of the matrix $\mathbf{A}$ are positive and semisimple. If the spectrum of $\mathbf{A}$ contains a complex conjugate pair then the system loses stability dynamically (flutter). Characteristic curve is a dependence of an eigenvalue of the matrix $\mathbf{A}$ on a chosen parameter, say on p_1, while other $n-1$ parameters remain fixed. Characteristic curves of the stable system lie on the real plane. We define a functional of critical load p_1^* as a minimal value of the parameter $p_1 \geq 0$ at which flutter instability occurs and formulate the optimization problem

$$p_1^*(\mathbf{z}) \to \sup, \quad \mathbf{z} = [p_2, \ldots, p_n]^T \in U \subset R^{n-1}. \tag{1}$$

Solving this problem numerically (for example by gradient method) it is naturally to expect that the functional will change continuously as the vector $\mathbf{z}$ varies in the gradient direction and that the characteristic curves will remain separated from each other on the interval $0\leq p_1<p_1^*$. It turns out that due to change of parameters $p_2, \ldots, p_n$ any two characteristic curves of the stable system $\lambda_i(p_1)$, $\lambda_j(p_1)$, corresponding to positive simple eigenvalues λ_i, λ_j, $i \neq j$, may come together, merge at some point, and then overlap forming a circle of complex eigenvalues (flutter instability) on some range of the parameter p_1. This overlapping leads to discontinuity of the critical flutter load, which blocks optimization process. Note that since the publication of the work [3] this phenomenon was an insurmountable obstacle for solving the problems of optimization of nonconservative systems: all attempts to find an improving variation leading away from a point of discontinuity or to prove the extremality of the functional at this point were unsuccessful (see, for example, [3 – 7]). In the present paper explicit formulae describing overlapping phenomenon in n-parameter nonconservative systems are obtained. These formulae use information on a system only at the merging point and allow qualitative as well as quantitative analysis of behavior of characteristic curves near that point. With the use of the developed theory necessary optimality conditions for some nonsmooth problems of structural optimization are derived and effective and easy-to-use numerical procedures are constructed. Mechanical examples are considered in detail.

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