SUBCRITICAL FLUTTER IN THE PROBLEMS OF ACOUSTICS OF FRICTION

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Summary Linearized models of rotating elastic bodies of revolution possess a spectrum that forms a mesh in the plane 'frequency' versus 'gyroscopic parameter' with double semi-simple eigenfrequencies at the nodes. In contact with the friction pads, the rotating media, for example the singing wine glass or the squealing disc brake, start to vibrate due to subcritical flutter instability. In the present paper a sensitivity analysis of the spectral mesh is developed for explicit predicting the onset of the friction-induced instabilities. The key role of indefinite damping and non-conservative positional forces in the development and localization of the subcritical flutter is clarified. It is shown that the instability scenarios, revealed in the general two-dimensional case, can take place also in more complicated finite-dimensional and distributed models of rotating symmetric bodies in frictional contact.

SPECTRAL MESHES OF ROTATING ELASTIC BODIES OF REVOLUTION

In 1638 Galileo Galilei remarked that "a glass of water may be made to emit a tone merely by the friction of the finger-tip upon the rim of the glass ". Shortly after Rayleigh qualitatively described the onset of bending waves in the singing wine glass by the friction, applied in the circumferential direction, and pointed out the proximity of the main audible frequency of the glass to the one of the spectrum of its free vibrations, Sperry and Lanchester invented a disc brake [7]. Nowadays, disc brake squeal—in general, a sound with one dominant high frequency—is the primary subject of investigations in acoustics of friction of rotating elastic bodies of revolution [7].

The author of one of the first theories of squeal, Spurr [1], experimentally observed that a rotating wine glass sang when the dynamic friction coefficient was a decreasing function of the velocity [2]. Linearizing the system with the negative friction-velocity gradient produces an eigenvalue problem with an *indefinite* damping matrix. Effectively negatively damped vibration modes may lead to complex eigenvalues with positive real parts and cause flutter instability, placing the fall in the dynamic friction coefficient with increasing velocity among the main empirical reasons for disc brake squeal, categorized by Kinkaid et al [7]. Two more reasons are *non-conservative* positional forces and the *splitting of the frequency of the doublet modes* of the symmetric disc when a friction force was applied [7].

Due to gyroscopic splitting of the doublet modes to the pairs of simple eigenvalues corresponding to the forward and backward traveling waves, which propagate along the circumferential direction, double semi-simple eigenvalues originate again at non-zero angular velocities, forming the nodes of the *spectral mesh* [8] of the crossed frequency curves in the plane 'frequency' versus 'angular velocity', Fig 1(a),(h). When the speed of rotation exceeds the critical speed, then the backward wave travels slower than the disc rotation speed and appears to be traveling forward with the negative effective energy, while that of the forward and backward traveling waves is positive [3]. In the *subcritical* speed region all the crossings of the frequency curves correspond to the forward and backward modes of the same signature, while in the *supercritical* region there exist crossings formed by the reflected and forward modes of opposite signature. According to Krein's theory [3], under Hamiltonian perturbations the crossings in the subcritical region veer away into *avoided crossings* (stability), while in the supercritical region the crossings of the modes of opposite signature turn into the rings of complex eigenvalues—*bubbles of instability* [3]—leading to a *supercritical flutter* important in the high speed applications like circular saws and computer storage devices. In the problems of acoustics of friction a *subcritical flutter* is (un)desirable as a source of instability at low speeds, which can occur due to non-Hamiltonian perturbations [10]. The sensitivity analysis of the present contribution shows how the nodes of the spectral mesh, situated in the subcritical range, can serve as the "keyboard" of a rotating elastic body of revolution.

COLLAPSE OF DISSIPATION-INDUCED BUBBLES OF INSTABILITY IN THE SUBCRITICAL RANGE

Consider a non-dimensional equation of a non-conservative system originating as a two-mode approximation of the models of rotating elastic bodies of revolution in frictional contact after their linearization and discretization [4, 5, 6, 7, 9]

$$\ddot{\mathbf{x}} + (2\Omega\mathbf{G} + \delta\mathbf{D})\dot{\mathbf{x}} + ((\beta^2 - \Omega^2)\mathbf{I} + \kappa\mathbf{K} + \nu\mathbf{N})\mathbf{x} = 0, \tag{1}$$

where a dot over a symbol denotes time differentiation, $\mathbf{x} \in \mathbb{R}^2$, and $\mathbf{I}$ is the identity matrix. The real matrices $\mathbf{D} = \mathbf{D}^T$, $\mathbf{G} = -\mathbf{G}^T$, $\mathbf{K} = \mathbf{K}^T$, and $\mathbf{N} = -\mathbf{N}^T$ are related to dissipative (damping), gyroscopic, potential, and non-conservative positional (circulatory) forces with magnitudes controlled by scaling factors δ, Ω, κ, and ν respectively; $\beta > 0$ is the frequency of free vibrations of the potential system when $\delta = \Omega = \kappa = \nu = 0$. Without loss of generality we assume $\det\mathbf{G} = \det\mathbf{N} = 1$. Separating time by setting $\mathbf{x}(t) = \mathbf{u}\exp(\lambda t)$ we arrive at the eigenvalue problem in λ. The operator $\mathbf{L}_0(\Omega) = \mathbf{I}\lambda^2 + 2\lambda\Omega\mathbf{G} + (\beta^2 - \Omega^2)\mathbf{I}$ of the unperturbed gyroscopic system has four eigenvalues $\lambda_p^\pm = i\beta \pm i\Omega$ and $\lambda_n^\pm = -i\beta \pm i\Omega$, forming the spectral mesh in the plane $(\Omega, \mathrm{Im}\lambda)$, Fig. 1(a). Two nodes of the mesh in the subcritical range $|\Omega| < \beta$ at $\Omega = \Omega_0 = 0$ correspond to the double semi-simple eigenvalues $\lambda = \pm i\beta$ with two orthogonal eigenvectors. Considering the splitting of the double eigenvalue $\lambda = i\beta$ under small perturbation of the gyroscopic

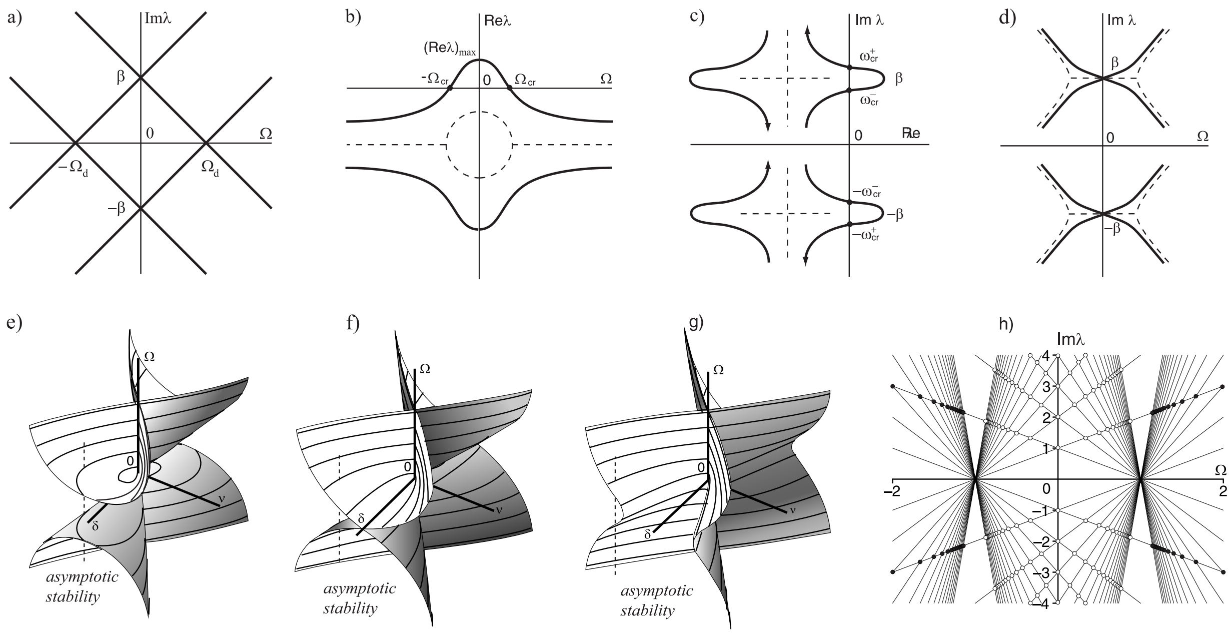

Figure 1. Deformation of the spectral mesh of a two-dimensional system (a)–(d); unfolding of the stability domain of the two-dimensional system with the change of the damping matrix from indefinite to positive definite (e)–(g); 30 modes of the spectral mesh of a rotating circular string [5] with the nodes in the sub- and supercritical regions marked by white and black dots respectively (h).

system $\mathbf{L}_0(\Omega) + \Delta\mathbf{L}(\Omega)$ with $\Delta\mathbf{L}(\Omega) = \delta\lambda\mathbf{D} + \kappa\mathbf{K} + \nu\mathbf{N} \sim \varepsilon$, where $\varepsilon = \|\Delta\mathbf{L}(0)\|$, we explicitly describe the deformation of the spectral mesh by small dissipative, non-conservative, and potential perturbations

$$\mathrm{Re}\lambda = -\frac{\mu_1+\mu_2}{4}\delta \pm \sqrt{\frac{|c|+\mathrm{Re}c}{2}}, \quad \mathrm{Im}\lambda = \beta + \frac{\rho_1+\rho_2}{4\beta}\kappa \pm \sqrt{\frac{|c|-\mathrm{Re}c}{2}}, \tag{2}$$

$$\mathrm{Re}c = \left(\frac{\mu_1-\mu_2}{4}\right)^2\delta^2 - \left(\frac{\rho_1-\rho_2}{4\beta}\right)^2\kappa^2 - \Omega^2 + \frac{\nu^2}{4\beta^2}, \quad \mathrm{Im}c = \frac{\Omega\nu}{\beta} - \delta\kappa\frac{2\mathrm{tr}\mathbf{KD} - \mathrm{tr}\mathbf{K}\mathrm{tr}\mathbf{D}}{8\beta}, \tag{3}$$

where $\mu_{1,2}$ and $\rho_{1,2}$ are the eigenvalues of the matrices $\mathbf{D}$ and $\mathbf{K}$, respectively. In the vicinity of the "keys" of the "keyboard" damping creates eigenvalue bubbles, shown in Fig. 1(b) by the dashed lines, which are dangerous by the ability to get positive real parts in presence of non-conservative positional forces or even without them, if the damping is indefinite. As is seen in Fig. 1(b)–(d), the activated and collapsed bubbles of instability yield the subcritical flutter of a rotating medium, forcing it to vibrate at a frequency $\omega_{cr}^- < \omega < \omega_{cr}^+$ and at the angular velocity $\Omega^2 < \Omega_{cr}^2$, where

$$\Omega_{cr} = \delta\frac{\mathrm{tr}\mathbf{D}}{4}\sqrt{-\frac{\nu^2 - \delta^2\beta^2\det\mathbf{D}}{\nu^2 - \delta^2\beta^2(\mathrm{tr}\mathbf{D}/2)^2}}, \quad \omega_{cr}^{\pm} = \beta \pm \frac{\nu}{2\beta}\sqrt{-\frac{\nu^2 - \delta^2\beta^2\det\mathbf{D}}{\nu^2 - \delta^2\beta^2(\mathrm{tr}\mathbf{D}/2)^2}}. \tag{4}$$

The first of equations (4) approximates the boundary of the domain of asymptotic stability with a singularity at the origin, which unfolds when the damping matrix is changing from the indefinite to definite, as shown in Fig. 1(e)–(g).

The proposed approach provides guidance to a classification of dissipative and non-conservative perturbations by their ability to cause the subcritical flutter, which can be used for checking and correcting particular models of disc brakes and other rotating elastic bodies of revolution in frictional contact.

References

[1] Spurr R.T.: A theory of brake squeal. *Proc. Inst. Mech. Engrs* (AD) **1**:33–52, 1961.
[2] Spurr R.T.: The ringing of wine glasses. *Wear* **4**:150–153, 1961
[3] MacKay R.S.: Stability of equilibria of Hamiltonian systems. In *Nonlinear Phenomena and Chaos* (ed. S. Sarkar), Hilger, Bristol:254–270, 1986.
[4] Chan S.N., Mottershead J.E., Cartmell M.P.: Parametric resonances at subcritical speeds in discs with rotating frictional loads. *Proc. Inst. Mech. Engrs C* **208**:417–425, 1994.
[5] Yang L., Hutton S.G.: Interactions between an idealized rotating string and stationary constraints. *J. Sound Vibr.* **185**(1):139–154, 1995.
[6] Xiong L.G., Chen H., Yi J.M.: Instability mechanism of a rotating disc subjected to various transverse interactive forces. *J. Mater. Proc. Techn.* **129**(1):534–538, 2002.
[7] Kinkaid N.M., O'Reilly O.M., Papadopoulos P.: Automotive disc brake squeal. *J. Sound Vib.* **267**: 105–166, 2003.
[8] Günther U., Kirillov O.N.: A Krein space related perturbation theory for MHD α^2-dynamos and resonant unfolding of diabolical points. *J. Phys. A: Math. Gen.* **39**(32): 10057–10076, 2006.
[9] von Wagner U., Hochlenert D., Hagedorn P.: Minimal models for disk brake squeal. *J. Sound Vibr.* **302**(3): 527–539, 2007.
[10] Kirillov O.N.: Destabilization paradox due to breaking the Hamiltonian and reversible symmetry. *Int. J. of Non-Lin. Mech.* **42**(1):71–87, 2007.